

# PERTURBED FLOER HOMOLOGY OF SOME FIBERED THREE MANIFOLDS II

ZHONGTAO WU

**ABSTRACT.** Modifying the method of [21], we compute the perturbed  $HF^+$  for some special classes of fibered three manifolds in the second highest  $\text{spin}^c$ -structures  $S_{g-2}$ . The special classes considered in this paper include the mapping tori of Dehn twists along a single non-separating curve and along a transverse pair of curves.

## 1. INTRODUCTION

Following [21], where the perturbed Heegaard Floer homology was defined and computed for the product three manifolds  $\Sigma_g \times S^1$ , we aim to modify our method for the computations of certain general fibered three manifolds. More precisely, we treat each fibered three manifold as a mapping torus  $M(\phi)$  for some orientation-preserving diffeomorphism  $\phi : \Sigma_g \rightarrow \Sigma_g$ , and decompose  $\phi$  into products of *Dehn twists*. The cases studied here consist of those Dehn twists along a single non-separating curve, and those along a transverse pair of curves.

Fibered three manifolds admit certain particularly simple “special Heegaard Diagrams”, first introduced by Ozsváth and Szabó in [12], where a genus  $2g+1$  Heegaard Diagrams was constructed for each mapping torus  $M(\phi)$ . Let  $S_k \subset \text{Spin}^c(M(\phi))$  denote the set of  $\text{spin}^c$ -structures  $\mathfrak{s}$  with  $\langle c_1(\mathfrak{s}), [\Sigma_g] \rangle = 2k$ . We will focus on the computation of the homology in the set of  $\text{spin}^c$ -structures  $S_{g-2}$ , for reasons to explain shortly. The details for the backgrounds of the “special Heegaard Diagram” are reviewed in section 2.

Section 3 through section 7 are dealing with various special classes of fibered three manifolds. All these cases are approached in a similar manner: We write down a special Heegaard diagram for each manifold, and find all the generators in  $S_{g-2}$ . The Euler characteristic is subsequently computed in each case, being made possible due to Lemma 3.1 and 3.2, where it is identified with the *Lefschetz number*  $L(\phi) := \sum_i (-1)^i \text{trace}(\phi_* : H_i(M) \rightarrow H_i(M))$ , and hence reduces the computation to a simple matter of linear algebra. The number of the generators and the Euler characteristic are found to be equal in each case, so we follow with the argument of [21]. In the end, the homology is found to be equal to the Euler characteristic in each  $\text{spin}^c$ -structure.

Results of a similar nature have been obtained by various other people. Seidel [19] considered the *symplectic Floer homology* of surface symplectomorphisms, calculating it for arbitrary compositions of Dehn twists along a disjoint collection curves. Eftekhary [4] generalized Seidel’s work to Dehn twists along two disjoint forests. It is then Cotton-Clay who achieved a vast generalization to include all pseudo-Anosov mapping class and reducible mapping class. Jabuka and Mark [5], on the other hand, computed the unperturbed results for the *Heegaard Floer homology*. Their results agreed with the previously mentioned papers wherever applicable, presenting a strong piece of favorable evidence for the conjectural existence of the isomorphisms between all versions of Floer homologies. Very recently, Taubes

[20] claimed a proof for the equivalence between *Seiberg-Witten Floer cohomology* and the *embedded contact homology*, though the equivalences of other versions of Floer homology are not established yet. Our paper is largely motivated by this, for the  $\text{spin}^c$ -structures in  $S_{g-2}$  are the relevant parts of the Heegaard Floer homology in the conjecture.

For the effects of the perturbations in Heegaard Floer theory are still poorly understood at the time being, the results in our paper are also intended as interesting examples for studying and understanding the relation. We find our perturbed homologies in section 4 agree with Jabuka and Mark's unperturbed results [5], while they are strictly smaller than the latter counterparts in section 3 and 5. It was also mentioned that all the homologies in this paper are identical to the corresponding Euler characteristics. Though it is perhaps too bold to conjecture such a phenomenon occurs for all (fibered) manifolds, we strongly believe it should hold for a much larger class of manifolds than those being discovered in this article. It may worth the effort to explore further in this direction.

**Acknowledgment.** I am much obliged to my advisor, Zoltán Szabó, for his continual encouragements and preparing me with the necessary backgrounds. I am also grateful to Joshua Greene and Yi Ni for helpful discussions at various points.

## 2. REVIEW OF THE SPECIAL HEEGAARD DIAGRAM

In this section, we review the special Heegaard Diagram construction by Ozsváth and Szabó [12, section 3]. Figure 1 is the Heegaard Diagram for  $\Sigma_g \times S^1$  used in [21], consisting of two  $2g$ -gons with standard identifications of edges and two punctured holes. They represent two genus  $g$  surfaces, joined together through the pairs of holes to make a genus  $2g + 1$  surface. All the  $\alpha$ 's and  $\beta$ 's curves are drawn along with their intersection points marked. We list all the interesting properties of the Heegaard Diagram:

- each  $\alpha_i \cap \beta_i$  twice, denoted by  $L_i$  and  $R_i$  respectively,  $1 \leq i \leq 2g$ .
- $\alpha_i \cap \beta_j = \emptyset$ , when  $i \neq j$ ,  $1 \leq i, j \leq 2g$ .
- $\alpha_{2g+1} \cap \beta_i$  twice, denoted by  $A_i$  and  $A'_i$  respectively,  $1 \leq i \leq 2g$ .
- $\alpha_i \cap \beta_{2g+1}$  twice, denoted by  $B_i$  and  $B'_i$  respectively,  $1 \leq i \leq 2g$ .

Next, we enumerate all generators in this Heegaard diagram. We sort them according to their  $\text{Spin}^c$  structures:

- When  $k \geq g$ ,  $S_k$  is empty.
- When  $k = g - 1$ ,  $S_{g-1}$  consists of a pair of generators:  $(A_{2g}, B_{2g}, L_1, L_2, \dots, L_{2g-1})$  and  $(A_{2g-1}, B_{2g-1}, L_1, \dots, L_{2g-2}, L_{2g})$ .
- When  $k = g - 2$ ,  $S_{g-2}$  consists of  $(2g - 1)$  pairs of generators:
 
$$a_1 := (A_{2g}, B_{2g}, R_1, L_2, \dots, L_{2g-1}),$$

$$a_2 := (A_{2g}, B_{2g}, L_1, R_2, \dots, L_{2g-1}),$$

$$\dots$$

$$a_{2g-2} := (A_{2g}, B_{2g}, L_1, L_2, \dots, R_{2g-2})$$

and

$$b_1 := (A_{2g-1}, B_{2g-1}, R_1, L_2, \dots, L_{2g}),$$

$$b_2 := (A_{2g-1}, B_{2g-1}, L_1, R_2, \dots, L_{2g}),$$

$$\dots$$

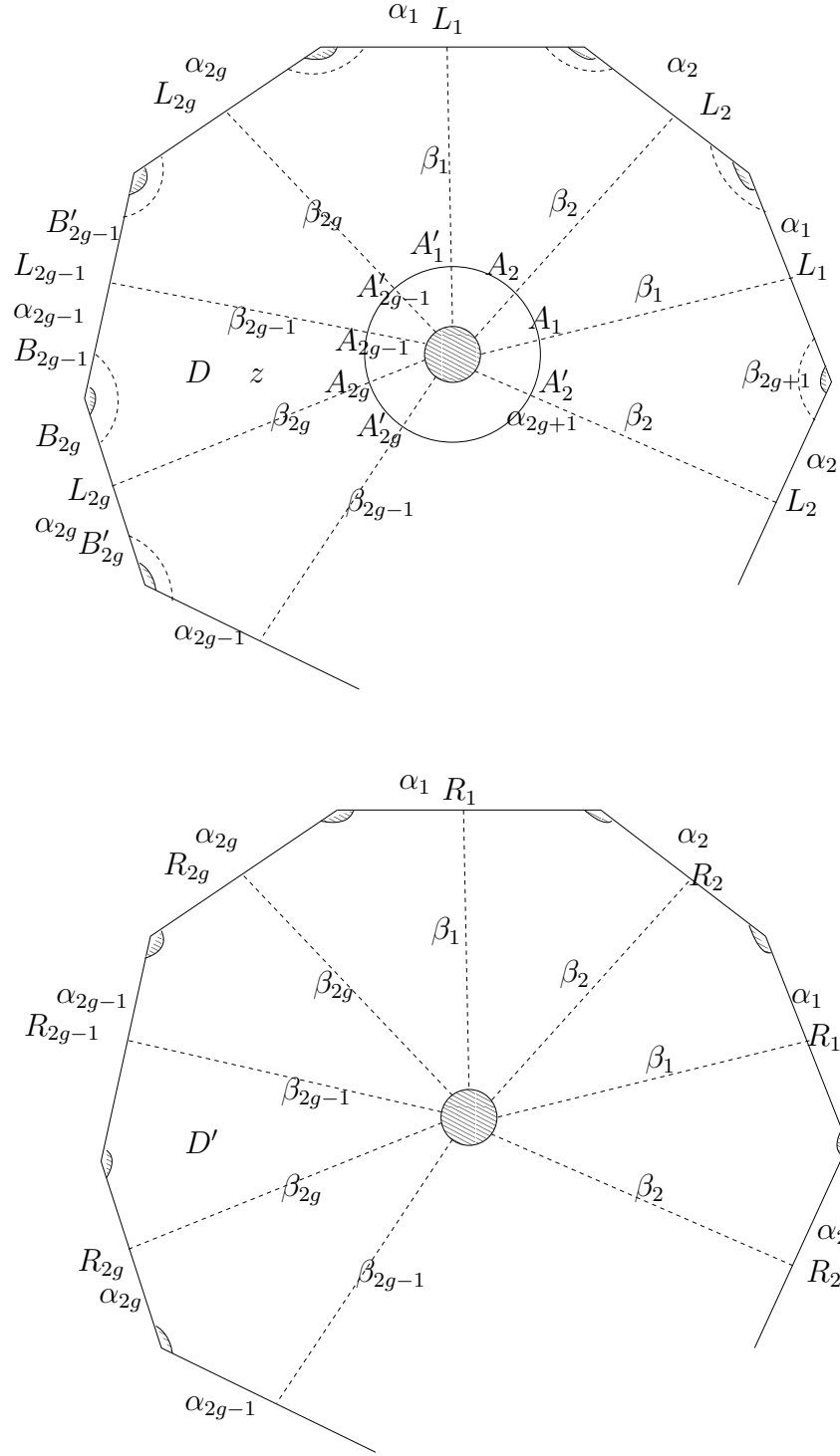


FIGURE 1. The special Heegaard Diagram for  $\Sigma_g \times S^1$ .

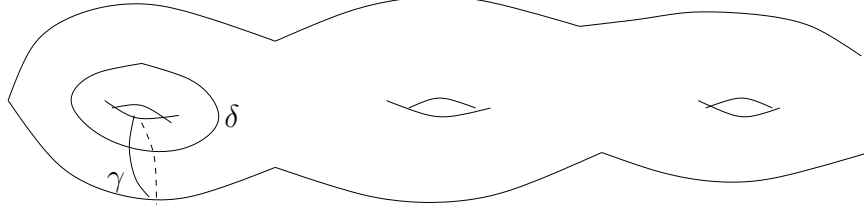


FIGURE 2.

$$b_{2g-2} = (A_{2g-1}, B_{2g-1}, L_1, L_2, \dots, R_{2g-2})$$

and

$$\begin{aligned} a_0 &:= (A_{2g}, B_{2g}, L_1, L_2, \dots, R_{2g-1}) \\ b_0 &:= (A_{2g-1}, B_{2g-1}, L_1, L_2, \dots, R_{2g}). \end{aligned}$$

Since  $a_0$  and  $b_0$  are connected by a disk  $D'$  not containing the basepoint  $z$ , we do not expect them to survive in the homology, and consequently we call them *fake generators*. The remaining  $(2g-2)$  pairs, on the other hand, are called *essential generators*.

- When  $0 < k < g-1$ ,  $S_k$  consists of  $\binom{2g-1}{g-1-k}$  pairs of generators: Simply replace  $(g-1-k)$  of  $L_i$  by  $R_i$  in the coordinates of the two generators of  $S_{g-1}$ . Among them,  $\binom{2g-2}{g-2-k}$  pairs are fake and  $\binom{2g-2}{g-1-k}$  pairs are essential.

In this way, we organize all the generators of the Heegaard diagram systematically. Such a schematic presentation of generators is as well available for a general fibered three manifold  $M(\phi)$ , and remains relatively simple for  $S_{g-2}$ .

Throughout the paper,  $g$  is implicitly assumed to be greater than 2.

### 3. MULTIPLE DEHN TWISTS ALONG A NON-SEPARATING CURVE

By a standard classification result in surface, any simple non-separating curve can be mapped to the standard position, namely  $\gamma$  in Figure 2, by a suitable surface automorphism. Hence, we may assume our manifolds to be  $M(t_\gamma^n)$  without loss of generality, where  $t_\gamma$  denotes the right-handed Dehn twist along  $\gamma$ .

Next, we draw the special Heegaard diagram of  $M(t_\gamma^n)$ . In general, for arbitrary  $M(\phi)$ , the  $\alpha$ 's and  $\beta$ 's curves inside the left-hand-side  $2g$ -gon are the same as that of  $\Sigma_g \times S^1$ . As for the right-hand-side  $2g$ -gon, whereas the  $\alpha$ 's curves are unaltered, the  $\beta$ 's curves are twisted according to  $\phi$ . Therefore, we would only exhibit the right-hand-side  $2g$ -gon of the Heegaard diagram, in which all information of the manifolds are encoded.

We proceed to enumerate all the generators in the set of the  $\text{spin}^c$ -structures  $S_{g-2}$  in the Heegaard diagram (Figure 3). Observe that a Dehn twist along  $\gamma$  does not introduce any new intersections between  $\alpha_i$  and  $\beta_i$ . Consequently, there is no additional generator, other than the  $(2g-1)$  pairs we initially had for  $\Sigma_g \times S^1$ .

Recall the following identity for the Euler characteristic of  $HF^+$  [21, Proposion 2.3]:

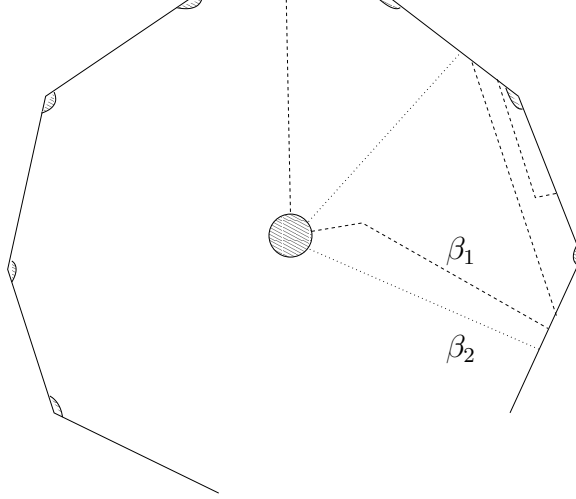


FIGURE 3. The Heegaard Diagram for  $M(t_\gamma^n)$ , when  $n = 2$ .

**Lemma 3.1.** *When  $\mathfrak{s}$  is a non-torsion  $\text{Spin}^c$  structure,  $HF^+(Y, s; \eta)$  is finitely generated, and the Euler characteristic*

$$\chi(HF^+(Y, \mathfrak{s}; \eta)) = \chi(HF^+(Y, s)) = \pm \tau_t(Y, \mathfrak{s}),$$

where  $\tau_t$  is Turaev's torsion function, with respect to the component  $t$  of  $H^2(Y; \mathbb{R}) - 0$  containing  $c_1(\mathfrak{s})$ .

Turaev's torsion function, derived from some complicated group rings over the CW-complex, is in general rather hard to compute. In our case for fibered three manifolds though, it is remarkably related to the Lefschetz numbers by the following identity [18],[6].

**Lemma 3.2.** *If we denote  $\tau_t(M(\phi), k)$  for the sum of all Turaev's torsion functions over the set of the  $\text{spin}^c$ -structures  $S_k$ , then*

$$\tau_t(M(\phi), k) = L(S^{g-1-k}\phi).$$

where the latter is the Lefschetz number of the induced function of  $\phi$  over the symmetric product  $S^{g-1-k}\Sigma_g$ . In particular, for  $k = g - 2$ ,

$$\tau_t(M(\phi), g - 2) = L(\phi).$$

Hence, applying Lemma 3.1 and 3.2, we have  $\chi(HF^+(M(t_\gamma^n), \mathfrak{s}_{g-2}; \omega)) = L(t_\gamma^n) = 2 - 2g$ . Following the argument of [21, section 4], we conclude:

**Theorem 3.3.**  $HF^+(M(t_\gamma^n), S_{g-2}; \omega) = \mathbb{A}^{2g-2}$ .

It is interesting to compare our result to those of the unperturbed Heegaard Floer homology [5] and the symplectic Floer homology [19]:  $HF^+(M(t_\gamma), \mathfrak{s}_{g-2}) \cong \mathbb{Z}_{(g-1)}^{2g-1} \oplus \mathbb{Z}_{(g)}$ ,  $HF^*(t_\gamma^n) \cong H^*(\Sigma_g, \gamma)$ . We find the ranks of our perturbed homology are diminished by 2, a typical phenomenon to anticipate from our previous experiences with computing  $\Sigma_g \times S^1$ .

We also remark that a generalization is readily attainable for the mapping torus  $M(t_{\gamma_1}^{n_1} \dots t_{\gamma_k}^{n_k})$  of the composition of a sequence of Dehn twists along some mutually disjoint curves  $\gamma$ 's in "standard" positions, thus covering all the cases considered in [19]. The techniques and results are exactly the same.

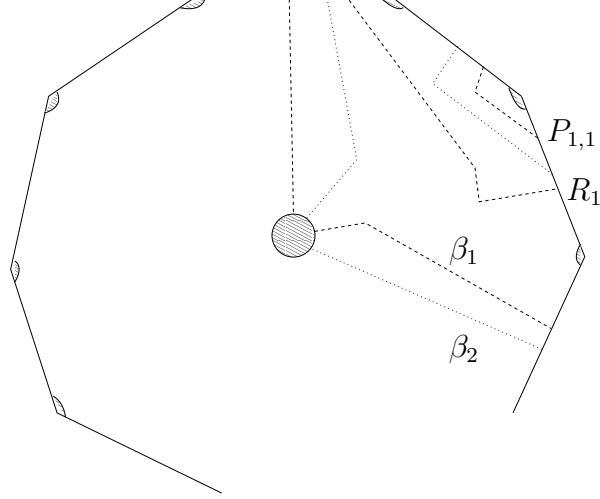


FIGURE 4. The Heegaard Diagram for  $M(t_\gamma^m t_\delta^n)$ , when  $m = 1, n = -1$ . Here,  $\beta_1$  is represented by the dashed curve, while  $\beta_2$  is represented by the dotted curve.

#### 4. MULTIPLE DEHN TWISTS ALONG A TRANSVERSE PAIR OF CURVES, CASE $m \cdot n < 0$

As before, we may assume the manifolds of our concern to be  $M(t_\gamma^m t_\delta^n)$ , without loss of generality, for  $\gamma, \delta$  in the standard positions of figure 2.

Consider the case  $m \cdot n < 0$  in this section, we have the Heegaard Diagram in Figure 4. We note extra intersections between  $\alpha_1, \alpha_2$  and  $\beta_1, \beta_2$ , and hence have more generators in  $S_{g-2}$ .

To be more precise, denote the  $|mn|$  extra intersection between  $\alpha_1$  and  $\beta_1$  by  $P_{i,j}$ , where  $1 \leq i \leq |m|$  and  $1 \leq j \leq |n|$ . Then, there are  $(2g - 1 + |mn|)$  pairs of generators in  $S_{g-2}$ , among which  $(2g - 2 + |mn|)$  are essential:

$$\begin{aligned} & (A_{2g}, B_{2g}, R_1, L_2, \dots, L_{2g-1}), \\ & (A_{2g}, B_{2g}, L_1, R_2, \dots, L_{2g-1}) \\ & \dots \\ & (A_{2g}, B_{2g}, L_1, L_2, \dots, R_{2g-1}) \end{aligned}$$

and

$$\begin{aligned} & (A_{2g-1}, B_{2g-1}, R_1, L_2, \dots, L_{2g}) \\ & (A_{2g-1}, B_{2g-1}, L_1, R_2, \dots, L_{2g}) \\ & \dots \\ & (A_{2g-1}, B_{2g-1}, L_1, L_2, \dots, R_{2g}) \end{aligned}$$

and

$$\begin{aligned} & (A_{2g}, B_{2g}, P_{i,j}, L_2, \dots, L_{2g-1}) \\ & (A_{2g-1}, B_{2g-1}, P_{i,j}, L_2, \dots, L_{2g}). \end{aligned}$$

We compute the Lefschetz number of  $L(t_\gamma^m t_\delta^n)$  from its definition

$$L(\phi) = \sum_i (-1)^i \text{trace}(\phi_* : H_i(M) \rightarrow H_i(M))$$

Both  $t_\gamma$  and  $t_\delta$  act trivially on  $H_0(\Sigma_g)$ ,  $H_2(\Sigma_g)$ , and  $2g-2$  of the basis of  $H_1(\Sigma_g)$ ; In the subspace for the remaining two basis of  $H_1(\Sigma_g)$ , their actions are represented by  $\begin{pmatrix} 1 & 1 \\ & 1 \end{pmatrix}$  and  $\begin{pmatrix} 1 & \\ -1 & 1 \end{pmatrix}$  respectively. We then find the trace of the matrices  $\begin{pmatrix} 1 & m \\ & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & \\ -n & 1 \end{pmatrix}$  and carry out the appropriate alternating sum. In the end, the Lefschetz number is found to be  $(2 - 2g + mn)$  - identical to the number of pairs of essential generators in this case.

The remaining arguments are similar. The rank of the homology is at least the Euler characteristic, being the same as the Lefschetz number, but simultaneously cannot exceed the number of pairs of essential generators. That leaves no other possibility than  $2g-2+|mn|$ . So we have:

**Theorem 4.1.**  $HF^+(M(t_\gamma^m t_\delta^n), S_{g-2}; \omega) = \mathbb{A}^{2g-2+|mn|}, \quad m \cdot n < 0.$

Again, compare the result with Theorem 5.7 of Jabuka and Mark [5]; The ranks agree this time.

## 5. MULTIPLE DEHN TWISTS ALONG A TRANSVERSE PAIR OF CURVES, CASE $m \cdot n > 0$

In this section, we compute the Heegaard Floer homology for the manifolds  $M(t_\gamma^m t_\delta^n)$  where  $m \cdot n > 0$ . By symmetry, it is enough to consider the case  $m, n > 0$ .

We have the following Heegaard diagram (Figure 5), and it can be subsequently simplified to Figure 6 by an isotopy on  $\beta_1$ . Note that the intersections  $R_1$  and  $P_{m,n}$  disappear there. Within such a Heegaard diagram, there are  $2(2g-4+mn)$  generators of  $S_{g-2}$ :

$$(A_{2g}, B_{2g}, L_1, R_2, \dots, L_{2g-1})$$

...

$$(A_{2g}, B_{2g}, L_1, L_2, \dots, R_{2g-1})$$

and

$$(A_{2g-1}, B_{2g-1}, L_1, R_2, \dots, L_{2g})$$

...

$$(A_{2g-1}, B_{2g-1}, L_1, L_2, \dots, R_{2g})$$

and

$$(A_{2g}, B_{2g}, P_{i,j}, L_2, \dots, L_{2g-1}), (i, j) \neq (m, n)$$

$$(A_{2g-1}, B_{2g-1}, P_{i,j}, L_2, \dots, L_{2g}), (i, j) \neq (m, n).$$

In the present case, it is possible and necessary to further partition all generators of  $S_{g-2}$  according to their Chern classes. Recall the first Chern class formula [8, section 7.1]:

$$\langle c_1(\mathfrak{s}_y), [\mathcal{P}] \rangle = \chi(\mathcal{P}) - 2\bar{n}_z(\mathcal{P}) + 2 \sum_{p \in y} \bar{n}_p(\mathcal{P}).$$

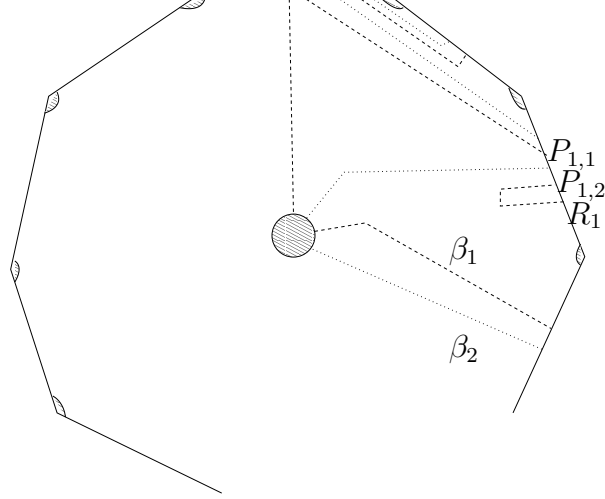


FIGURE 5. The Heegaard Diagram for  $M(t_\gamma^m t_\delta^n)$ , when  $m = 1, n = 2$ .  $\beta_1$  is represented by the dashed curve, while  $\beta_2$  is represented by the dotted curve.

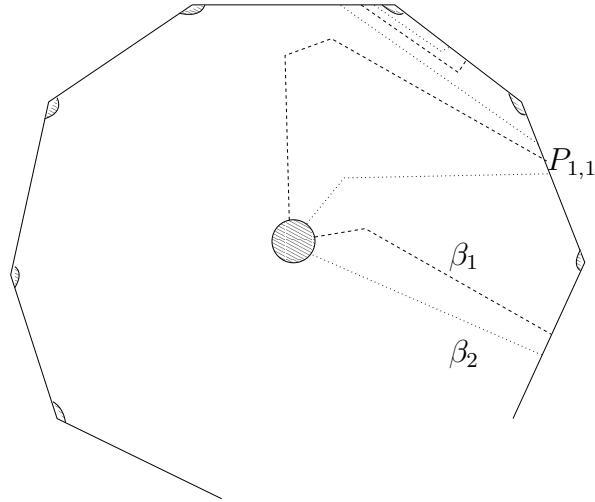


FIGURE 6. The simplified Heegaard Diagram after isotopying on  $\beta_1$ .

where  $\mathfrak{s}_y$  is a  $\text{spin}^c$ -structure corresponding to  $y$ . Applying this formula, we find  $(A_{2g}, B_{2g}, P_{i,j}, L_2, \dots, L_{2g-1}), (A_{2g-1}, B_{2g-1}, P_{i,j}, L_2, \dots, L_{2g}), (i, j) \neq (m, n)$  lie on  $mn - 1$  different  $\text{spin}^c$ -structures, denoted by  $\mathfrak{s}_{i,j}$  respectively; While all the remaining generators  $(A_{2g}, B_{2g}, L_1, R_2, \dots, L_{2g-1}), \dots, (A_{2g}, B_{2g}, L_1, L_2, \dots, R_{2g-1}), (A_{2g-1}, B_{2g-1}, L_1, R_2, \dots, L_{2g}), \dots, (A_{2g-1}, B_{2g-1}, L_1, L_2, \dots, R_{2g})$  lie on the other  $\text{spin}^c$ -structure, denoted by  $\mathfrak{s}_{m,n}$ .

For each  $\text{spin}^c$ -structure  $\mathfrak{s}_{i,j}, (i, j) \neq (m, n)$ , there are exactly two generators  $(A_{2g}, B_{2g}, P_{i,j}, L_2, \dots, L_{2g-1}), (A_{2g-1}, B_{2g-1}, P_{i,j}, L_2, \dots, L_{2g})$  and an obvious holomorphic disk  $D$  connecting them. The argument from [21, section 3] for 3-torus can thus be applied and shows

$$HF^+(M(t_\gamma^m t_\delta^n), \mathfrak{s}_{i,j}; \omega) = \mathbb{A}.$$



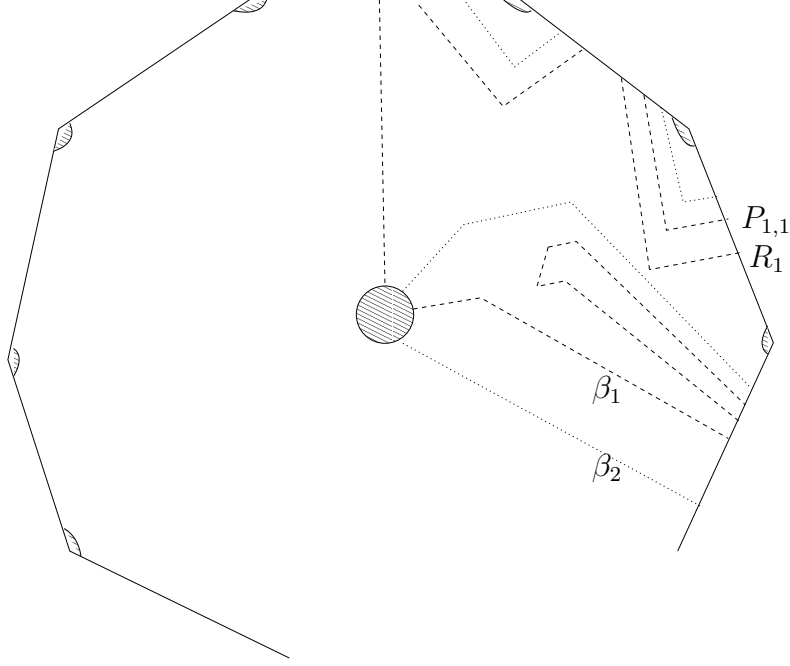


FIGURE 7. The Heegaard Diagram of  $M(t_\gamma t_\delta t_\gamma)$ . An isotopy on  $\beta_1$  can be carried out to cancel the pairs of intersection points  $R_1$  and  $P_{1,1}$ .

To determine the homology in the  $\text{spin}^c$ -structure  $\mathfrak{s}_{m,n}$ , note its Euler characteristic is:

$$\begin{aligned} \chi(HF^+_{\mathfrak{s}_{m,n}}) &= \tau_t(2g - 2) - \sum_{(i,j) \neq (m,n)} \chi(HF^+_{\mathfrak{s}_{i,j}}) \\ &= 2 - 2g + mn - (mn - 1) \\ &= 3 - 2g. \end{aligned}$$

There are also exactly  $2g - 3$  pairs of essential generators, so we can repeat the argument within  $\mathfrak{s}_{m,n}$ , as before, and conclude

$$HF^+(M(t_\gamma^m t_\delta^n), \mathfrak{s}_{m,n}; \omega) = \mathbb{A}^{2g-3}.$$

Putting everything together, we have:

**Theorem 5.1.**  $HF^+(M(t_\gamma^m t_\delta^n), S_{g-2}; \omega) = \mathbb{A}^{2g-4+mn}$ ,  $m \cdot n > 0$ .

Again, compare the result with Theorem 5.3 of Jabuka and Mark [5]; our rank is smaller by two.

## 6. MULTIPLE DEHN TWISTS ALONG A TRANSVERSE PAIR OF CURVES, CASE III

The manifolds considered here have the form  $M(t_\gamma^{m_1} t_\delta^{n_1} t_\gamma^{m_2})$ , where  $m_1, m_2, n_1 > 0$ . The Heegaard diagram for the case  $m_1 = n_1 = m_2 = 1$  is drawn in Figure 7. An isotopy can be carried out for  $\beta_1$  to remove the intersections  $R_1$  and  $P_{1,1}$ . Such an isotopy is available in general, so that we would have  $2g - 4 + (m_1 + m_2)n_1$  pairs of essential generators in the simplified Heegaard diagram.

The number of  $\text{spin}^c$ -structures in  $S_{g-2}$  is  $(m_1 + m_2)n_1$ , and these  $\text{spin}^c$  structures are denoted by  $\mathfrak{s}_{i,j}$ . Similar to the case of section 5, there exists exactly one pair of essential generators in each  $\mathfrak{s}_{i,j}$  for  $(i, j) \neq (m_1 + m_2, n_1)$ , and  $2g - 3$  pairs of essential generators in the remaining distinguished  $\text{spin}^c$ -structure  $\mathfrak{s}_{m_1+m_2, n_1}$ .

Hence, for all  $(i, j) \neq (m_1 + m_2, n_1)$ ,

$$HF^+(M(t_\gamma^{m_1} t_\delta^{n_1} t_\gamma^{m_2}), \mathfrak{s}_{i,j}; \omega) = \mathbb{A}.$$

Since the Lefschetz number is  $2 - 2g + (m_1 + m_2)n_1$ , we have:

$$\begin{aligned} \chi(HF_{\mathfrak{s}_{m_1+m_2, n_1}}^+) &= \tau_t(2g - 2) - \sum_{(i,j) \neq (m_1+m_2, n_1)} \chi(HF_{\mathfrak{s}_{i,j}}^+) \\ &= 2 - 2g + (m_1 + m_2)n_1 - ((m_1 + m_2)n_1 - 1) \\ &= 3 - 2g \end{aligned}$$

The usual argument applies once more and shows:

$$HF^+(M(t_\gamma^{m_1} t_\delta^{n_1} t_\gamma^{m_2}), \mathfrak{s}_{m_1+m_2, n_1}; \omega) = \mathbb{A}^{2g-3}.$$

Putting all the  $\text{spin}^c$ -structures together, we conclude with:

**Theorem 6.1.**  $HF^+(M(t_\gamma^{m_1} t_\delta^{n_1} t_\gamma^{m_2}), S_{g-2}; \omega) = \mathbb{A}^{2g-4+(m_1+m_2)n_1}.$

## 7. MULTIPLE DEHN TWISTS ALONG A TRANSVERSE PAIR OF CURVES, CASE IV

Lastly, we consider the manifolds of the form  $M(t_\gamma^{m_1} t_\delta^{n_1} \dots t_\gamma^{m_k} t_\delta^{n_k})$ , where  $m_i \cdot n_j < 0$ . In other words, they are the mapping tori of Dehn twists along  $\gamma$  and  $\delta$  with alternating signs.

We compute the Lefschetz number. If we denote the trace of the following matrix  $\begin{pmatrix} 1 & m_1 \\ & 1 \end{pmatrix}.$

$\begin{pmatrix} 1 & \\ -n_1 & 1 \end{pmatrix} \dots \begin{pmatrix} 1 & m_k \\ & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & \\ -n_k & 1 \end{pmatrix}$  by  $T$ , then the Lefschetz number is  $4 - 2g - T$ .

Meanwhile, there are  $2g + T - 4$  pairs of essential generators. This can be evidently seen by relating the intersections of  $\alpha_i$  and  $\beta_i$  to the trace of the matrix  $\begin{pmatrix} 1 & |m_1| \\ & 1 \end{pmatrix}.$

$$\begin{pmatrix} 1 & \\ |n_1| & 1 \end{pmatrix} \dots \begin{pmatrix} 1 & |m_k| \\ & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & \\ |n_k| & 1 \end{pmatrix}.$$

In our case, the number of pairs of essential generators is, once again, exactly the Lefschetz number. Hence, we repeat our argument and conclude:

**Theorem 7.1.**  $HF^+(M(t_\gamma^{m_1} t_\delta^{n_1} \dots t_\gamma^{m_k} t_\delta^{n_k}), S_{g-2}; \omega) = \mathbb{A}^{2g-4+T} \quad m_i \cdot n_j < 0.$

## REFERENCES

- [1] **Y Ai, Y Ni**, *Two applications of twisted Floer homology*, preprint, available at arXiv:0809.0622
- [2] **Y Ai, T Peters**, *The twisted Floer homology of torus bundles*, preprint, available at arXiv:0806.3487
- [3] **A Cotton-Clay**, *Symplectic Floer homology of area-preserving surface diffeomorphisms*, preprint, available at arXiv:0807.2488
- [4] **E Eftekhary**, *Floer homology of certain pseudo-Anosov maps*, J. Symplectic Geom. 2 (2004), no. 3, 357–375
- [5] **S Jabuka, T Mark**, *Product formulae for Ozsváth–Szabó 4-manifolds invariants*, to appear in Geom. Topol., available at arXiv:0706.0339
- [6] **M Hutchings, Y. Lee**, *Circle valued Morse theory, Reidemeister torsion, and Seiberg–Witten invariants of 3-manifolds*, Topology 38 (1999), no. 4, 861–888

- [7] **P Ozsváth, Z Szabó**, *Holomorphic disks and topological invariants for closed three-manifolds*, Ann. of Math.(2), 159 (2004), no. 3, 1027–1158
- [8] **P Ozsváth, Z Szabó**, *Holomorphic disks and three-manifold invariants: properties and applications*, Ann. of Math.(2), 159 (2004), no. 3, 1159–1245
- [9] **P Ozsváth, Z Szabó**, *Holomorphic triangle invariants and the topology of symplectic four-manifolds*, Duke Math. J. 121 (2004), no. 1, 1–34
- [10] **P Ozsváth, Z Szabó**, *Absolutely Graded Floer homologies and intersection forms for four-manifolds with boundary*, Adv. Math. 173 (2003), no. 2, 179–261
- [11] **P Ozsváth, Z Szabó**, *Holomorphic disks and knot invariants*, Adv. Math. 186 (2004), no. 1, 58–116
- [12] **P Ozsváth, Z Szabó**, *Heegaard Floer homology and contact structures*, Duke Math. J. 129 (2005), no. 1, 39–61.
- [13] **P Ozsváth, Z Szabó**, *Holomorphic disks and genus bounds*, Geom. Topol. 8 (2004), 311–334 (electronic)
- [14] **P Ozsváth, Z Szabó**, *On the Heegaard Floer homology of branched double-covers*. Adv. Math. 194 (2005), no. 1, 1–33
- [15] **P Ozsváth, Z Szabó**, *Holomorphic triangles and invariants for smooth four-manifolds*, Adv. Math. 202 (2006), no. 2, 326–400.
- [16] **P Ozsváth, Z Szabó**, *Knot Floer homology and integer surgeries*, Algebr. Geom. Topol. 8 (2008), 101–153 (electronic)
- [17] **J Rasmussen**, *Floer homology and knot complements*, PhD Thesis, Harvard University (2003), available at arXiv:math.GT/0306378
- [18] **D Salamon**, *Seiberg-Witten invariants of mapping tori, symplectic fixed points, and Lefschetz numbers*, Turkish J. Math. 23 (1999), no. 1, 117–143
- [19] **P Seidel**, *The symplectic Floer homology of a Dehn twist*, Mathematical Research Letters 3 (1996) 829–834
- [20] **C Taubes**, *Embedded contact homology and Seiberg-Witten Floer cohomology I*, preprint, available at arXiv:0811.3985
- [21] **Z Wu**, *Perturbed Floer homology of some fibered three manifolds*, preprint, available at arXiv:0809.4697